647510 **MSC4MatC403x** Seat No :________

M.Sc. (Maths.) Semester - 4 (*CBCS*) Examination
March/April - 2024 (NEW COURSE)
Number Theory -2 (Core)

Time: 02:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q. 1 (A) Answer any Two out of Three. **[10]**

(1) There is a polynomial $f_n(x)$ of degree n, leading coefficient 1 with integer coefficient such that $f_n(2\cos\theta) = 2\cos n\theta$.

(2) If $\frac{p}{q}$ and $\frac{r}{s}$ are consecutive fractions in any row, then among all rational fractions with values between these two show that $\frac{(p+r)}{(q+s)}$ is the unique fraction with smallest denominator.

(3) If θ is an irrational number then there are integers $a_0, a_1, \dots, a_n, \dots$ such that $a_i \geq 1;\ \forall\, i = 1,2,3,4, \dots$ and $\theta = < a_0,\ a_1, \dots, a_{n,} \dots\ >$.

Q. 1 (B) Answer any One out of Two. **[04]**

(1) Prove that $\sqrt{2} + \sqrt{3}$ is a root of $x^4 - 10x^2 + 1 = 0$, and hence establish that is irrational.

(2) Suppose (1) $\lim\limits_{n\to\infty} r_{2n} = \theta$, (2) $\lim\limits_{n\to\infty} r_{2n-1} = \theta'$ then $\theta = \theta'$.

Q. 2 (A) Answer any Two out of Three. **[10]**

(1) Show that the value of any infinite continued fraction is an irrational number.

(2) If θ is an irrational number then $\exists$ infinitely many rational $\frac{a}{b}$ such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{\sqrt{5}b^2}$; $b >\ 0$ and $(a, b) = 1$.

(3) Show that any rational number can be written as a finite simple continued fraction.

Q. 2 (B) Answer any One out of Two. **[04]**

(1) Expand $\frac{\sqrt{5}+1}{2}$ as an infinite continued fraction.

(2) Expand the rational fraction $29/51$ into finite simple continued fraction.

Q. 3 (A) Answer any Two out of Three. **[10]**

(1) If p, q is a positive solution of $x^2 - dy^2 = 1$ then $\frac{p}{q}$ is a convergent of the continued fraction expansion of $\sqrt{d}$.

(2) Prove that π is an irrational number.

(3) If θ is a quadratic irrational number then the continued fraction expansion of θ is purely periodic iff (1) $\theta > 1$, (2) $-1 < \theta' < 0$. Where θ' is conjugate of θ.

Q. 3 (B) Answer any One out of Two. **[04]**

(1) Show that $\sqrt{2}$ is a quadratic irrational number.

(2) If x is a real number and if $x +\ x^{-1} < \sqrt{5}$ then (1) $x < \frac{\sqrt{5}+1}{2}$, (2) $x^{-1} > \frac{\sqrt{5}-1}{2}$.

Q. 4 (A) Answer any Two out of Three. **[10]**

(1) Find all solution of $30x - 21y = 54$.

(2) Prove that the radius of the inscribed circle of a Pythagorean triangle is always an integer.

(3) If u and v are $\geq 1, (u, v) = 1$ and if uv is perfect square number then u is perfect square number and v is perfect square number.

Q. 4 (B) Answer any One out of Two. **[04]**

(1) If x, y, z is primitive Pythagorean triple then show that one of the integers $x\ or\ y$ is even while the other is odd.

(2) (1) What is the Diophantine equation?
(2) Write down the difference between primitive Pythagorean triple and Pythagorean triple.

Q. 5 (A) Answer any Two out of Three. **[10]**

(1) If $< a_0,\ a_1, \ldots, a_j > = < b_0, b_1, \ldots, b_n >$ where these finite continued fractions are simple, and if $a_j > 1, and\ b_n > 1$, then
j = n and $a_i = b_i$ for all $i = 1,2,3, \ldots, n$.

(2) Find first three positive solution of $x^2 - 2y^2 = 1$.

(3) If (x, y, z) is Pythagorean triple then
gcd (x, y, z) = gcd (x, y) = gcd (z, y) = gcd (x, z).

Q. 5 (B) Answer any One out of Two. **[04]**

(1) If $x_0,\ y_0$ is a positive solution of the equation $x^2 - dy^2 = 1$ then prove that $x_0 > y_0$.

(2) Find the value of $< 2,1,2,1,2,1, \ldots >$

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